MHD waves and turbulence in the expanding solar wind

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SHINE Conference 2019, August 4th
Outline

- Structure of the solar wind
- Basics of turbulence: Kolmogorov’s theory
- Solar wind turbulence
  - Observations
  - Equation & Kraichnan’s theory
  - Methods for study of solar wind turbulence
- Expanding-Box-Model simulations
- Conclusion
Solar Wind

- Model of the wind *far from* the sun:
  - Velocity is almost radial: \( \mathbf{U} = U \hat{e}_r \)
  - Already accelerated within 0.2-0.3 AU: \( U = \text{Const} \) beyond.
  - Spiral magnetic field: \( \mathbf{B} = B_r \hat{e}_r + B_\phi \hat{e}_\phi \)
  - Expansion effect:
    - \( n \propto r^{-2} \)
    - \( T \propto r^{-a} \) with \( a \leq 1 \)
    - \( B_r \propto r^{-2} \)
    - \( B_\phi \propto r^{-1} \)
  - Alfvén point \( r_c \):
    - \( U(r_c) = \left( \frac{B_r}{\sqrt{\mu_0 \rho}} \right)(r_c) \)

![Graph showing speed vs. radial distance]

**Alfvén point**
Solar Wind

- **Fast wind**
  - Origin: coronal holes (high latitude)
  - Fast speed: $> 600$ km/s
  - Low density
  - High temperature

- **Slow wind**
  - Origin: streamer belt (low latitude)
  - Slow speed: $300 - 500$ km/s
  - High density
  - Low temperature
Solar Wind

Stream interaction

Illustration of the corotating-interaction region
Basics on turbulence

The most important concepts in turbulence study

- **Nonlinear interaction:**
  - Leads to energy transfers between scales
  - \( \mathcal{F}_k(A \cdot \nabla B) = \int dp \left[ i (k - p) \cdot A_p \right] B_{k-p} \)

- **Scales:**
  - Energy-containing scales \( l_e \): Energy is *injected only at these scales*
  - Energy-dissipation scales \( l_d \): Energy is *dissipated only at these scales*
  - Inertial scales \( l \): Energy is *neither injected nor dissipated but flows through scales (cascade)*
    - \( l_d \gg l \gg l_e \)

- **Energy transfer (dissipation) rate:**
  - \( \varepsilon \): Energy dissipation/transfer rate per unit mass
  - Dimension: \( v^2/\tau \)
  - Constant throughout the inertial range
Basics on turbulence

- Energy (power) spectrum density
  - Definition: \( E(k) = |\hat{v}(k)|^2 dk \), where \( \hat{v}(k) = \mathcal{F}[v(x)] = \frac{1}{2\pi} \int dx \, v(x) e^{-ikx} \)
  - Estimate of the perturbation amplitude on scale \( k \): \( v(k) \sim \sqrt{kE(k)} \)
  - Scaling of \( E(k) \) in inertial range is determined by the \textit{nonlinear interactions}

- Example: non(weakly)-magnetized fluid
  - \( \varepsilon \sim \frac{v^2(k)}{\tau_{NL}(k)} \)
  - From N-S equation: \( \frac{\partial v}{\partial t} \sim v \frac{\partial v}{\partial x} \rightarrow \frac{v}{\tau_{NL}} \sim k v^2 \rightarrow \tau_{NL}(k) \sim \frac{1}{kv(k)} \)
  - \( \varepsilon \sim k v^3 \rightarrow E(k) \sim \varepsilon^2 k^{-5/3} \)
Observations

Early observations

Feature of Turbulence

Coleman, 1968

Russell, 1972

Belcher and Davis, 1971

2: Russell, 1972, Comments on the measurement of power spectra of the interplanetary magnetic field
3: Belcher and Davis, 1971, JGR, 76, 16
Modern views

- The solar wind turbulence is most-of-the-time Alfvénic:
  - Nearly-constant density and $|B|$ (incompressible)
  - Dominated by outward-propagating Alfvén waves
  - Inward-propagating Alfvén waves are also present but weaker
- Nonlinear interaction between outward/inward waves is needed
- Convective structures also exist in the solar wind, especially the slow wind
- Questions:
  - Where and how are the outward/inward waves generated?
    - Outward waves may have solar origin
    - Inward waves cannot have solar origin but may be generated by sheared streams, instabilities, etc.
  - Why the dominance of the outward waves decays with radial distance?
  - How is the imbalance between magnetic and kinetic energies produced?
  - The compressible MHD turbulences?
  - Anisotropy of the turbulence?
Basic equation

Scale-separated, incompressible, MHD equation
- \( \rho = \rho_0, \mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1, \mathbf{b} = \mathbf{b}_0 + \mathbf{b}_1 \)
- Elsässer variables: \( Z^\pm = \mathbf{u}_1 \pm \frac{b_1}{\sqrt{4\pi\rho}} \)
- (momentum equation) \( \pm \) (induction equation/\( \sqrt{4\pi\rho} \))

\[
\frac{\partial Z^\pm}{\partial t} + (\mathbf{u}_0 \mp \mathbf{V}_A) \cdot \nabla Z^\pm + Z^\mp \cdot \nabla (\mathbf{u}_0 \pm \mathbf{V}_A) \mp (Z^\pm \cdot \nabla \ln \sqrt{\rho}) \mathbf{V}_A \mp [(\mathbf{u}_0 \pm \mathbf{V}_A) \cdot \nabla \ln \sqrt{\rho}] \frac{Z^+ - Z^-}{2} = NL^\pm
\]

\( NL^\pm \sim Z^\mp \cdot \nabla Z^\pm \)

- Propagation
- inhomogeneous background: linear evolution and coupling
- Nonlinear interaction
Kraichnan Spectrum\(^1\): \(E(k) \sim k^{-\frac{3}{2}}\)

- Alfvénic turbulence with strong background field \(B_0\)
- \(B_0\) weakens the nonlinear interaction
  
  - \(\tau_A = l/V_{A0}\)
  - \(\varepsilon \propto \tau_A\)
  - Dimensional analysis
    - Write \(\varepsilon \sim \tau_A z^\alpha k^\beta\) with \(\alpha, \beta\) to be determined
    - \(\varepsilon\) has dimension \(v^3 k\)
    - \(\alpha = 4, \beta = 2\)
    - Substitute \(z = \sqrt{kE(k)}\)
    - \(E(k) \sim (\varepsilon V_{A0})^{\frac{1}{2}} k^{-\frac{3}{2}}\)

1: Kraichnan, 1965, Physics of Fluids, 8, 1385
Two important parameters

- **Normalized cross helicity**
  \[ \sigma_c = \frac{|z^-|^2 - |z^+|^2}{|z^-|^2 + |z^+|^2} = -\frac{2u \cdot v_A}{|u|^2 + |v_A|^2} \]
  - Relative abundance of the energies of outward/inward Alfvén waves

- **Normalized residual energy**
  \[ \sigma_r = \frac{|u|^2 - |v_A|^2}{|u|^2 + |v_A|^2} = \frac{2z^+ \cdot z^-}{|z^+|^2 + |z^-|^2} \]
  - Relative abundance of the kinetic/magnetic energies

\[ z^- = u_1 - \frac{b_1}{\sqrt{4\pi\rho}} \text{: outward wave} \]
\[ z^+ = u_1 + \frac{b_1}{\sqrt{4\pi\rho}} \text{: inward wave} \]
Methods to study solar wind turbulence

- In-situ measurements
- Model\(^1,2\)
  - Derive equations for the averaged second-order moments of \(z^\pm\),\(\langle |z^\pm|^2 \rangle\) & \(\langle |z^+ \cdot z^-| \rangle\), which evolve on large (stream) scales
- Direct Numerical Simulation (DNS)

2: Tu et al., 1984, JGR 89:A11
Direct Numerical Simulation

- Solve the full MHD equation (or two fluid, kinetic, etc.)
- Solar wind is spherically expanding and is huge
- Impossible to simulate the whole heliosphere
- Expanding Box Model (EBM)
  - A rectangular box moving with the radial mean flow
  - Angular size of the box is constant
  - Neglect curvatures and assume cartesian coordinates locally

\[
U = U_0 + u = U_r \hat{e}_r + u
\]

\[
U_x \approx u_x + U_0
\]

\[
U_y \approx u_y + U_0 \frac{y}{R(t)}
\]

\[
U_z \approx u_z + U_0 \frac{z}{R(t)}
\]

Expanding Box Model

- Translation and normalization: 
  \[ x' = x - R(t), \quad y' = \frac{R_0}{R(t)} y, \quad z' = \frac{R_0}{R(t)} z \]

- Periodic boundary conditions
Expanding Box Model

- Corotating coordinates
  \[
  \begin{pmatrix}
  x' \\
  y' \\
  z'
  \end{pmatrix} = \begin{pmatrix}
  \cos \alpha & -\sin \alpha & 0 \\
  \sin \alpha & \cos \alpha & 0 \\
  0 & 0 & 1
  \end{pmatrix} \begin{pmatrix}
  \tilde{x} \\
  \tilde{y} \\
  \tilde{z}
  \end{pmatrix}
  \]

- Able to simulate the compression region:
  - Initial fast-slow stream structure (see the 1D run):
    - \( B_0 = B_0 \hat{e}_x', u_0 = u_0(y')\hat{e}_x, T_0(y'), \rho_0(y') \)

- Parameters
  - \( R_0 = 30R_s, L_x = 10R_s, L_y = \pi R_0 \): half circle
  - initial spiral angle \( \alpha = 8.1\degree \), uniform \( |B| = 250 \text{ nT} \)
  - \( \rho = 5 \text{ nPa}, \kappa = 1.5 \)
  - \( U_r = 464 \text{ km/s} \)
  - Fast wind: 700 km/s, 140 cm\(^{-3}\)
  - Slow wind: 340 km/s, 360 cm\(^{-3}\)
Test: 1D simulation

- No waves are added
Initial condition: Alfvénic wave band

- Circularly-polarized Alfvén waves

\[
\begin{align*}
b_{1,o} &= \delta b \sum_{N=1}^{N_{max}} \frac{1}{\sqrt{N}} \left[ \cos \left( \frac{2\pi N}{L_{x'}} x' + \phi_{N,o} \right) \hat{e}_y' + \sin \left( \frac{2\pi N}{L_{x'}} x' + \phi_{N,o} \right) \hat{e}_z \right], \\
b_{1,i} &= r_{io} \times \delta b \sum_{N=1}^{N_{max}} \frac{1}{\sqrt{N}} \left[ \cos \left( \frac{2\pi N}{L_{x'}} x' + \phi_{N,i} \right) \hat{e}_y' + \sin \left( \frac{2\pi N}{L_{x'}} x' + \phi_{N,i} \right) \hat{e}_z \right],
\end{align*}
\]

- Initial energy spectrum: \( k_{x'}^{-1} \)
- \( N_{max} = 16 \)
- \( r_{io} \): in/out amplitude ratio
Outward-dominant $r_{io} = 0.2$

- Phase-mixing & depletion of wave energy

$Z$-component of the outward Elsässer variable $z_{out,z}$
Outward-dominant $r_{io} = 0.2$

$$E^T = \frac{1}{2} (|z^-|^2 + |z^+|^2)$$

$$\sigma_c = \frac{|z^-|^2 - |z^+|^2}{|z^-|^2 + |z^+|^2}$$

Total energy

Density fluctuation

Pure kinetic

Pure magnetic
Outward-dominant $r_{io} = 0.2$

Power spectra of $z$ along $y'$ corrected by $k_{y'}^{5/3}$

Power spectra of $z$ along $x'$ corrected by $k_{x'}^{5/3}$
Radial evolution of Elsässer energies

Outward-dominant

Run A

Outward-dominant

Run B

Balanced

Run C

Inward-dominant

Run D

Radial evolution of $E_{out}$ and $E_{in}$ corrected by $R/R_0$
Conclusion of the simulations

- Shear between fast and slow streams: depletes wave energy and decreases cross helicity
- Compression between streams: facilitates the process
- Kolmogorov-like spectrum is easier to form in the longitudinal direction (quasi-perpendicular) than in the parallel direction
- The radial evolution of the Alfvén wave energy is highly dependent on the regions in the stream structure.

SEE MY POSTER FOR MORE DETAILS
Summary

- The solar wind turbulence is most-of-the-time Alfvénic:
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Future reading


THANK YOU